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Roper Resonances as Vibrating Flux Tubes between Quarks

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The Roper resonances of the nucleon are described as transverse vibrations of a stretched flux tube between the three quarks. The proton is modeled using current mass quarks interacting with a confining linear flux tube potential plus the spin-dependent parts of a one-gluon exchange potential. The proton ground state has no vibrations and the confining flux tube has the minimum length required to connect the three quarks. The flux tube has a V or Y shape of two or three segments, depending on the locations of the quarks. The vibrations of the flux tube have nodes at the quarks and at the apex of the Y-shaped configuration and provide the vibrational excitation energy to describe the proton excitations. The amplitude of the transverse vibrations is found from a geometric analysis, and depends on the string constant of the flux tube potential. The Roper 1.440-GeV resonance energy is very nearly reproduced by the vibration with mode number 1 acting in only one segment of the flux tube. The vibration with mode number 2 in one segment of the flux tube closely reproduces the second proton excitation at 1.710 GeV. The Δ excited states are also well reproduced by these modes of a vibrating flux tube.

1. INTRODUCTION

The Roper resonance is an excited state of the proton that has an energy of about 1.440 GeV and the same quantum numbers as the $(1/2^+)$ proton, of rest energy 0.938 GeV. Other resonances with nucleon quantum numbers at higher energies have been identified [1] in phase shift analyses. The Roper resonance does not show up as a clear resonance in π nucleon scattering analyses, but only through the inelasticity of the P_{11} phase shift. The particle data group [1] shows its width as about 350 MeV. It is lower in energy than the 1.530–1.675 GeV negative-parity resonances. Attempts to describe the

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Roper resonance as a breathing mode or radial motion excitation of three quarks have not been particularly successful. Nonrelativistic harmonic oscillator models, constituent quark models [2–6], collective models, and current quark models [7–9] of the proton have all had varying partial successes in predicting the Roper resonance properties. The resonance has also been studied as a monopole excitation in the solution of the Nambu–Jona-Lasino soliton model [10] and as a possible radial node in the hyperradial wave function of the composite three-quark wave function [8]. Rather than an excited state of three quarks, Krehl and Speth have described the Roper as a quasibound resonance of two pions and a nucleon [11]. Brown *et al.*[12] used a three-quark model with an aharmonic collective oscillation of a confining bag surface to describe the Roper resonance.

Recent models of mesons and nucleons [2–4, 8, 13–15] use a linear flux tube potential to confine the quarks and antiquarks in mesons and nucleons. Isgur and Paton developed a hybrid meson model [16, 17] based on gluonic excitations involving an excited flux tube. The production and decay rates of such low-lying meson states have encouraging agreement with experiment [18]. Such gluonic excitations in the nucleon are modeled here as transverse vibrations of the flux tube connecting the quarks. This oscillation of the confining mechanism, the flux tube, is similiar in spirit to the oscillations of the bag surface considered by Brown *et al.* [12]. To confine the three quarks in the nucleon, the flux tube potential is characterized as $V = bS$, where b is the string constant, about 0.90 GeV/fm, and S is the length of the flux tube connecting the three quarks. The shape of the flux tube is either a two-segment V (1/4 of phase space) or a Y shape of three segments (3/4 of phase space), depending on the relative location [2, 19] of the three quarks. The quarks are at the ends of the Y-shaped flux tube, with no quark at its vertex. For the nucleon ground state, S has the minimum length, as the flux tube is composed of straight-line segments. S is longer than the lengths between the quarks for the excited states due to transverse vibrations set up in one or more segments.

The colored gluons exchanged between quarks in QCD interact with each other. This interaction causes the flux lines to parallel one another, rather than spread out as in QED. The interacting gluons form a flux tube of approximately uniform cross-sectional area, so that the potential energy becomes proportional to the flux tube length, which is tied to the distance between the quarks. The string constant b reflects this cross-sectional area and the energy density of the interacting exchanged gluons [13]. This potential is assumed to be a scalar. It can be combined with the spin-dependent magnetic part of the one-gluon exchange potential (OGEP) to explain [20] the proton–Δ rest-mass differences. A similiar linear confining potential, plus attractive OGEP terms, can explain most rest-frame properties of the proton [21].

Current quark masses are used, with parameters adjusted to reproduce the proton energy and rms charge radius. The magnetic moment and axial charge are also well reproduced in this model, with 10-MeV quark masses.

The quark dynamics is assumed described by the three-body Dirac equation solved in hypercentral approximation. The Jacobi coordinates properly handle the center of mass of the three quarks in the rest frame of the system. The center of mass of the three quarks is maintained at the origin. The flux tube potential is the modeled result of the exchange of many gluons, each of zero mass. The effect their energy has on the system center of mass is ignored. The nucleon rest energy E will be calculated using the composite three-quark wave function found from solving the three-body Dirac equation as an eigenvalue problem [22]:

$$\left\{\begin{array}{c}(\vec{\alpha}_1 \cdot \vec{p}_1 + m_1\beta_1) + (\vec{\alpha}_2 \cdot \vec{p}_2 + m_2\beta_2) \\ +(\vec{\alpha}_3 \cdot \vec{p}_3 + m_3\beta_3) + \beta_1\beta_2\beta_3 V \\ -(2\alpha_s/3)[(\vec{\alpha}_1 \cdot \vec{\alpha}_2/r_{12}) + (\vec{\alpha}_1 \cdot \vec{\alpha}_3/r_{13}) \\ +(\vec{\alpha}_2 \cdot \vec{\alpha}_3/r_{23})]\end{array}\right\}\Psi = E\Psi \tag{1}$$

Hyperspherical coordinates [23–25] are used, and the three-body Dirac equation is solved in hypercentral approximation. The hypercentral approximation limits the composite three-quark wave function to a single $(1/2^+)^3$ configuration for the proton. The basic idea is to use the chain rule of calculus to change the partial derivatives of the kinetic energy operator with respect to r_1, etc., into partial derivatives with respect to the hyperradius. The hyperspherical coordinates are the hyperradius r and five hyperangles. One possible set of these hyperangles is as follows. The three quark locations define a triangle, with the quarks at the corners. Any two internal angles are the first hyperangles. The triangle has a normal. The spherical polar angles defining its direction are the next two angles. The azimuthal orientation of the triangle about this normal is the fifth hyperangle. The hyperradius is

$$r^2 = 2(r_{12}^2 + r_{13}^2 + r_{23}^2)/3 = [r_{ij}^2 + r_k^2] \tag{2}$$

The Jacobi coordinates in the partition (ij, k) are

$$\vec{r}_k = \sqrt{3}(\vec{x}_k - \vec{X}), \qquad \vec{r}_{ij} = \vec{x}_i - \vec{x}_{\mathrm{j}} \tag{3}$$

where $\vec{X}$ is the center-of-mass coordinate and $\vec{x}_i$ are those of the particles.

2. HYPERCENTRAL APPROXIMATION VALIDITY FOR THE FLUX TUBE POTENTIAL

If the potential is independent of the hyperangles, then the hypercentral aproximation would be exact. V is the confining flux tube potential. The hyperangular dependence of the flux tube potential has been shown in Fabre De La Ripelle and Lassant [26], so it is not reproduced here. It is remarked there that the hypercentral approximation is a good one in a Schrödinger equation context. The validity of the hypercentral approximation for the flux tube potential in a relativistic three-quark context will now be probed. The flux tube potential is independent of spin, spherically symmetric, but not hyperspherically symmetric. The quark spins and the spin-dependent part of the one-gluon exchange potential are temporarily neglected. The hyperspherical expansion with the flux tube potential within the three-body Klein–Gordon equation context is considered. This is written as

$$[P^2 + (3m + V)^2 - E^2]\Psi = 0 \tag{4}$$

$\vec{P}$ is the six-dimensional vector momentum conjugate to the six hyperspherical coordinates of the system. The wave function is given by the sum over cyclic permutations c and the harmonic values K:

$$\Psi = \sum_{K,c} \Phi_K(\vec{r}_{ij}, \vec{r}_k) \tag{5}$$

The wave function has a definite total orbital angular momentum L_{total}, with projection M, both of which are zero for the ground state studied here.

The hypercentral approximation for the ground state includes the $K = K_{\min}$ component only with zero orbital angular momentum for each of the quarks. A $K_{\min+2}$ calculation is done, including additionally a component where two units of orbital angular momentum l_{ij} and l_k are assigned to the orbital angular momentum part of the wave function, with the combination coupled to a total angular momentum of zero. The expansion of the wave function is over K harmonics [24, 25], now truncated at $K = 2$. Each term in the expansion can be written as

$$\Phi_k = U_k(\Omega)R_k(r)/r^{5/2} \tag{6}$$

The K harmonic functions $U_k(\Omega)$ are orthonormalized. Upon doing the hyperangular integration over the hyperangles Ω, the flux tube potential term can be written as

$$\langle U_0|V|U_0\rangle = cr, \qquad \langle U_2|V|U_2\rangle = c_2 r, \qquad \langle U_2|V|\ U_0\rangle = Dr \tag{7}$$

where $c = 1.118b$, $c_2 = 1.18b$, and $D = -0.13b$. The Klein–Gordon equation, after integration over hyperangles, in $K_{\min+2}$ approximation becomes

$$\begin{aligned}[-d^2/dr^2 + L_0(L_0 + 1)/r^2 + (3m + cr)^2 - E^2]R_0 \\ = -D[6mr + (c + c_2)r^2]R_2\end{aligned} \tag{8}$$

$$[-d^2/dr^2 + L_2(L_2 + 1)/r^2 + (3m + c_2 r)^2 - E^2]R_2$$
$$= -D[6mr + (c + c_2)r^2]R_0 \qquad (9)$$

L_0 is 3/2 for the $K_{\min}$ term and L_2 is 7/2 for the $K_{\min+2}$ term. To get an analytic solution, we solve these equations in perturbation theory, first neglecting the coupling D term, and the linear $6mr$ terms. The solutions to the unperturbed equations are then Gaussians. The unperturbed energies are $E_{00}^2 = 9m^2 + 6c$ and $E_{20}^2 = 9m^2 + 10c_2$. The unperturbed radial wave functions are $R_0 = N_0 r^{5/2}\exp(-cr^2)$ and $R_2 = N_2 r^{9/2}\exp(-c_2 r^2)$. The coupled equations are then solved perturbatively including the coupling D terms and the linear $6mr$ terms using the unperturbed wave functions. We are interested in solutions where the quark mass m is small, but report results for masses up to 0.3 GeV. The eigenenergy of the resulting two by two matrix is determined as is the contribution to the norm of the $K_{\min+2}$ terms. Neglecting spin, the three-quark model of the nucleon should be compared to the average of the proton and delta masses, 1.085 GeV. The flux tube potential constant is set to 0.9 GeV/fermi in agreement with meson studies. The hypercentral energy E_{00} is 1.057 GeV for massless quarks. The contribution to the norm for the $K_{\min+2}$ terms is 0.031 for massless quarks. The eigenenergy of the coupled equations is 1.043 GeV for massless quarks. As the assumed quark

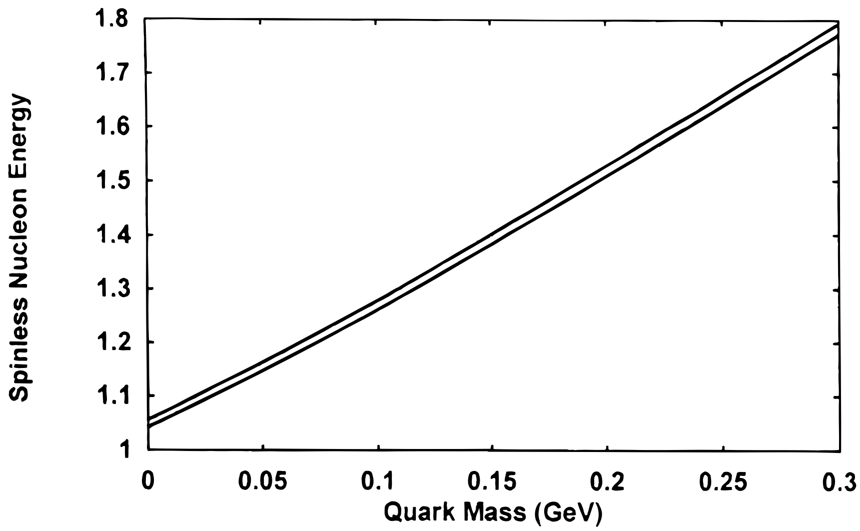

Fig. 1. Ground state eigenenergy of the three-body Klein–Gordon equation in $K_{\min+2}$ approximation using the flux tube potential. The upper curve is the hypercentral approximation eigenenergy. The lower curve is the more accurate $K_{\min+2}$ eigenenergy. The energy difference is about 15 MeV.

mass increases, the predicted energies increase, as can be seen in Fig 1. The shift of energy (a decrease) from including the $K_{\min+2}$ term is about 14–20 MeV, depending on the quark mass assumed. The contribution to the norm of the $K_{\min+2}$ state is about 3–5% as shown in Fig 2. When quark masses of about 12 MeV are used, the $K_{\min+2}$ eigenenergy matches the expected 1.085 GeV. This Klein–Gordon approach using small-mass quarks provides reasonable eigenenergies for the spinless nucleon, and shows that the hypercentral approximation for the linear flux tube potential is rather good in this calculation neglecting spins. This ends the discussion of the Klein–Gordon equation in $K_{\min+2}$ approximation and the neglect of spin. We return to the hypercentral approximation.

3. FLUX TUBE VIBRATIONS

Including spin for the quarks, the composite three-quark wave function is labeled by the J and parity quantum numbers of the upper component of the orbitals occupied by the three quarks. The quarks are assumed to be in a $(1/2^+)^3$ configuration coupled to a total spin of 1/2 for the proton and a spin of 3/2 for the Δ. The space, spin, color, and flavor dependences of the three-quark wave function for the proton and for all excited states considered here are the same as in ref. 8. There is a totally antisymmetric color wave

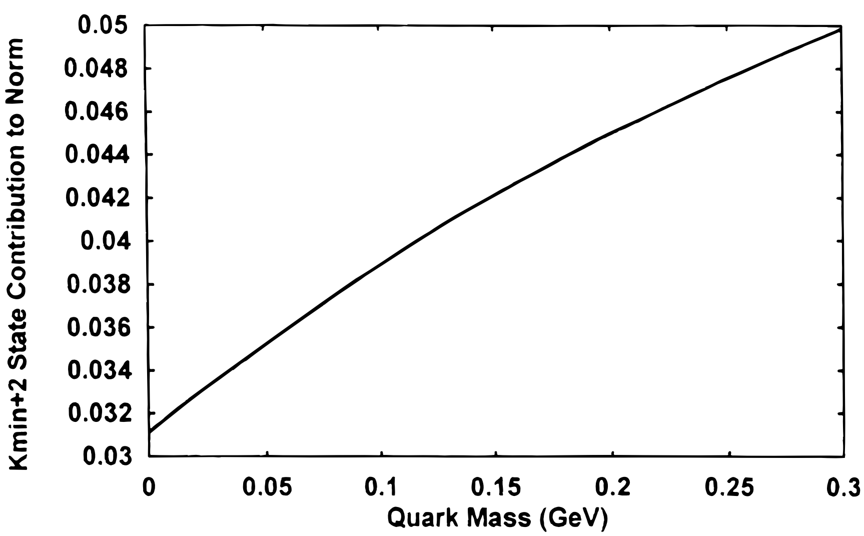

Fig. 2. The $K_{\min+2}$ component contribution to the norm of the ground-state wave function versus quark mass. Quark masses of about 15 MeV are realistic in this modeling.

function. The flavor, orbital angular momentum, and spin combined parts of the wave function are totally symmetric. The configuration labels are the total angular momentum and parity of the upper component of each quark wave function. The upper components have no orbital angular momentum for the configuration considered. The lower components have one unit of orbital angular momentum, which couples to the spin of one-half, to form the total angular momentum of one-half for each quark. With small quark masses assumed, the lower components are not small, as they contribute more than 0.4 to the normalization [18].

After color and hyperangular integration, the three-body Dirac equation reduces to a set of eight coupled first-order differential equations to solve for the eight components of the composite three-quark wave function [27, 28]. With equal-mass quarks and for a configuration where all quarks have the same set of spatial quantum numbers, symmetries reduce the problem to four coupled first-order differential equations involving four unknown components of the composite three-quark wave function. These equations can be solved as an eigenvalue problem using a power series solution [19], where recursive relations are developed for the power series coefficients.

In the nucleon ground state, the flux tube model consists of straight-line segments of minimum total length connecting the three quarks. The flux tube potential is proportional to this length. Any quantum mechanical zero-point energy of vibrations about these segments or of motion of the apex connecting the segments is incorporated into the energy associated with the flux tube length, $V = bS$. The excitations of this system are described as transverse vibrations of the flux tube segments between the quarks in the nucleon. These vibrations are now described.

Consider a distance r_i from the origin to one of the quarks. Placing one node at the origin and a quark along the x axis, and the transverse vibration in the xy plane, we have that the vibration is characterized by the standing wave form

$$y = A\sin[n\pi x/r_i] \tag{10}$$

This waveform is the shape of the vibrating flux tube at the time of maximum displacement from the equilibrium straight-line segment position when the flux tube is momentarily at rest during the vibration cycle. The amplitude A of such a vibrational mode can be determined by invoking the string potential constant b and requiring that the potential be proportional to the flux tube segment length L. The segment length L is

$$L = \int_0^{r_i} [1 + (dy/dx)^2]^{1/2}\, dx \tag{11}$$

Using the waveform, this becomes

$$L = \int_0^{r_i} [1 + (n\pi A/r_i)^2 \cos^2(n\pi x/r_i)]^{1/2} \, dx \tag{12}$$

When the amplitude A is small compared to the quark distance from the origin, r_i, the segment length is

$$L = \int_0^{r_i} [1 + 0.5(n\pi A/r_i)^2 \cos^2(n\pi x/r_i)] \, dx = r_i + (n\pi A)^2/(4r_i) \tag{13}$$

The contribution to the potential of the vibrating segment is

$$V_{\text{seg}} = br_i + b(n\pi A)^2/4r_i \tag{14}$$

The last term is the extra energy associated with the flux tube transverse vibration. As the gluons are massless, the deBroglie wavelength determined from $p = h/\lambda$ implies a vibrational energy of

$$E_{\text{vib}} = hc/\lambda \tag{15}$$

The vibrations are restricted to having nodes at the quark locations, and also at the Y vertex, if that is the shape of the flux tube. This restricts the wavelength to

$$\lambda = 2r_i/n \tag{16}$$

and so the extra energy of the vibrating flux tube is

$$E_{vib} = n\pi\hbar c/r_i \tag{17}$$

where n is the mode number (integer). For the vibrational mode of fewest nodes, n is one. Comparing coefficients to the energy of a vibrating string, the flux tube vibration amplitude is given by

$$A^2 = 4\hbar c/bn\pi \tag{18}$$

This amplitude is independent of r_i for large r_i and is taken as independent of r_i for all separations r_i. Then the flux tube length of a vibrating segment can be expressed in terms of the mode number n and the string constant b as

$$L(n, r_i) = \int_0^{r_i} [1 + (4\hbar cn\pi/br_i^2) \cos^2(n\pi x/r_i)]^{1/2} \, dx \tag{19}$$

The flux tube length L from equation (12) as a function of r_i is shown in Fig. 3 for mode numbers 1 and 2. The flux tube segment is vibrating about its equilibrium position of minimum length. L is the segment length when the vibrating flux tube is momentarily at rest, as it is briefly during each vibration cycle. For a mode number of zero, the flux tube segment length L is equal to r_i. The extra energy associated with a vibrating flux tube segment in modes 1 and 2 in this sinusoidal wave form model is very similiar to the hybrid gap energy determined quantum mechanically by treating the flux tube as a chain of massive beads [29].

To insure the wave function has the proper symmetry upon exchange of any pair of quarks, the vibrating flux tube potential is written as the cyclic sum over the three permutations of the Jacobi variables as

$$V(n_1, n_2) = (b/3) \sum_c (L(n_1, r_i) + L(n_2, r_j) + r_k) \tag{20}$$

This treatment for the flux tube length fixes the apex of the flux tube to be at the origin, rather than at the location where the flux tube length is the minimum distance connecting the three quarks. To the extent that this length can be characterized as a quadratic (see Fig. 3) the vibrating flux tube length of the symmetrized potential can be written as

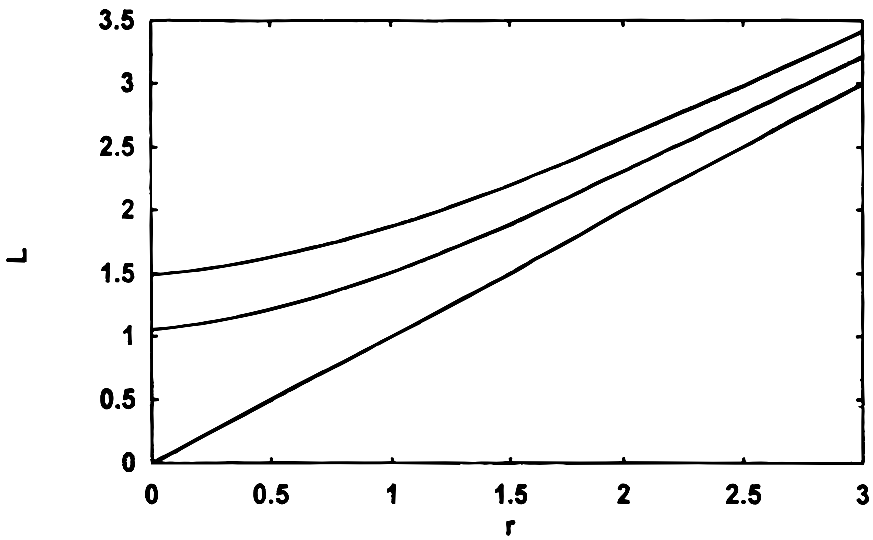

Fig. 3. Flux tube segment length L for a transverse vibrating mode versus r. This is the vibrating flux tube length at the instant when it is momentarily at rest during each cycle when it is vibrating. The flux tube potential is assumed proportional to this length. The lower curve is for mode number 1, the upper curve is for mode number 2.

$$L_s = [(Ar_1^2 + Br_1 + C) + (Ar_2^2 + Br_2 + C) + (Ar_3^2 + Br_3 + C)]/3 \tag{21}$$

where the coefficients A, B, and C depend on the mode numbers of the vibration. Now the constant and the quadratic terms in this vibrating flux tube length are both independent of hyperangle. The hypercentral approximation is exact for these terms. The linear terms containing B as a factor are also well approximated by the hypercentral approximation, just as the linear flux tube potential has been shown to be. Thus the hypercentral approximation for this vibrating flux tube potential is as valid as for the linear straight-line-segment flux tube potential.

The three body Dirac equation is solved numerically with this vibrational excitation potential included in addition to the magnetic part of the one-gluon exchange potential used to reproduce the proton and Δ energy separation. The eigenenergies, and their comparison to experiment [1] are shown in Table I for various modes assumed for the vibrations of the flux tube.

The string potential constant, the quark mass (10 MeV), and the magnetic part of the one-gluon exchange potential are able to reproduce the proton and the Δ ground-state rest energies, with the flux tube potential at its minimum length. The excited states come from transverse vibrations of the flux tube about its minimum length location in this model.

The eigenenergy for the Roper resonance is found to be 1.430 GeV, in very good agreement with the experimental value 1.440 GeV [1]. Good agreement with experiment is also found the first and second excited states of the spin-1/2 and spin-3/2 nucleons. The third excited state at 2.10 GeV is well reproduced in the proton. The corresponding 2.27-GeV state predicted for the Δ is not seen in the phase shift analyses.

4. SUMMARY

The proton is modeled as three quarks of small current quark mass interacting with a scalar linear confining potential with an additional OGEP

Table I. Hypercentral Eigenenergies

Mode numbers	$J = 1/2$		$J = 3/2$	
n_1, n_2	Calc.	Exp.	Calc.	Exp.
0, 0	0.938	0.938	1.232	1.232
1, 0	1.430	1.440	1.610	1.600
2, 0	1.757	1.710	1.916	1.920
1, 1	2.103	2.100	2.270	Not seen

spin-dependent term. The three-body Dirac equation is solved in hypercentral approximation which restricts the wave function to a single configuration, chosen to be the $(1/2^{+})^{3}$. A transverse vibrational model of the confining flux tube is used to describe the excited states of the nucleon with the same quantum numbers as the ground state. For a given quark separation, the flux tube has more energy when it is vibrating about its minimum length location between the quarks than when the flux tube is motionless at its minimum length. The endpoints of the flux tube vibrations are the quarks, or the vertex of the Y-shaped flux tube. These are assumed to be nodes of the flux tube vibrations. The amplitude of a vibration is determined from geometrical considerations and is inversely related to the flux tube string constant. The lowest vibrational mode numbers, 0 (no vibration), 1, and 2, predict eigenenergies that correspond to the lowest observed states of the proton with energies 0.938, 1.430, and 1.746 GeV, respectively. This agrees well with the experimental values of 0.938, 1.440, and 1.710 GeV. When the mode number 1 vibration is simultaneously in two segments of the flux tube, the system energy is predicted to be 2.086 GeV. This is in close agreement with the resonance seen at 2.100 GeV. The energy of observed excited states of the Δ are also well reproduced in this vibrating flux tube model.

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